A Model Independent Approach to Semi-Inclusive Deep Inelastic Scattering [1]

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Abstract

We present a method for extraction of detailed information on polarized quark densities from semi-inclusive deep inelastic scattering $l + N \to l + h + X$, in both LO and NLO QCD without any assumptions about fragmentation functions and polarized sea densities. The only symmetries utilised are charge conjugation and isotopic spin invariance of strong interactions.

1. At present the possibility to obtain a full information on the parton helicity densities in a polarized nucleon is related to semi-inclusive deep inelastic scattering (SIDIS) experiments of polarized leptons on polarized nucleons:

$$\overrightarrow{e} + \overrightarrow{N} \to e + h + X, \qquad h = \pi^\pm,\, K^\pm, ... \tag{1}$$

The first polarized SIDIS measurements were done by the SMC [1] and HERMES [2] collaborations, where the asymmetry A_N^h was measured:

$$A_N^h = \frac{1 + (1-y)^2}{2y\,(2-y)}\,\frac{\Delta\sigma_N^h}{\sigma_N^h} = \frac{\sum e_q^2\,(\Delta q\, D_q^h + \Delta\bar q\, D_{\bar q}^h)}{\sum e_q^2\,(q\, D_q^h + \bar q\, D_{\bar q}^h)}, \qquad (LO). \tag{2}$$

Here $\Delta\sigma_N^h$ (σ_N^h) is the measured polarized (unpolarized) cross section. Though this asymmetry allows, in principle, a full extraction of the polarized parton densities, in practice, however, it faces two major problems. First, a good knowledge of the fragmentation functions (FFs) D_q^h is required. At present, these functions are rather poorly known and a number of model dependent assumptions are made. And second, it is not clear how to extend the "purity" method, used in the analysis in LO, to the case of NLO. As the fulfilled and planned measurements of polarized SIDIS are done at rather low Q^2, NLO corrections have to be taken into account.

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2. Here we show [3,4] that a measurement of the difference asymmetries $A_N^{h^+-h^-}$ and ratios $R_N^{h^+-h^-}$:

$$A_N^{h^+-h^-} = \frac{1+(1-y)^2}{2y\,(2-y)}\,\frac{\Delta\sigma_N^{h+} - \Delta\sigma_N^{h-}}{\sigma_N^{h^+} - \sigma_N^{h^-}}, \quad R_N^{h^+-h^-} = \frac{\sigma_N^{h+} - \sigma_N^{h-}}{\sigma_N^{DIS}}, \tag{3}$$

gives a lot of information on the polarized quark densities, in both LO and NLO, without any assumptions about the FFs. In general this can be traced as follows. C-invariance of strong interactions leads to $D_G^{h-\bar{h}} = 0$ and $D_q^{h-\bar{h}} = -D_{\bar{q}}^{h-\bar{h}}$, which immediatelly singles out in $A_N^{h^+-h^-}$ and $R_N^{h^+-h^-}$ quantities that are flavour non-singlets for both the parton densities and the FFs. This implies that: 1) the badly known gluons G, ΔG and $D_G^{h-\bar{h}}$ do not enter these asymmetries, 2) in its Q^2-evolution $A_N^{h^+-h^-}$ and $R_N^{h^+-h^-}$ do not mix with other quantities, apart from those entering them and, 3) $A_N^{h^+-h^-}$ can give information only on non-singlet parton densities: Δu_V, Δd_V, $\Delta\bar{u} - \Delta\bar{d}$, $\Delta s - \Delta\bar{s}$, etc, but in a model independent way. This holds in general and is true in any order in QCD. Depending on the SU(2) properties of the detected final hadron h, one obtains different pieces of information. This method for extracting the polarized quark densities will be used at Jefferson Lab. [5], in the planned experiment E04-113, Semi-SANE.

3. If $h = \pi^\pm$, SU(2) and C-invariance reduce the number of independent pion FFs: $D_u^{\pi^+-\pi^-} = -D_d^{\pi^+-\pi^-}$, $\quad D_s^{\pi^+-\pi^-} = 0$. Then in LO we have:

$$\begin{aligned} A_p^{\pi^+-\pi^-}(x,z,Q^2) &= \frac{4\Delta u_V - \Delta d_V}{4u_V - d_V}(x,Q^2) \\ A_n^{\pi^+-\pi^-}(x,z,Q^2) &= \frac{4\Delta d_V - \Delta u_V}{4d_V - u_V}(x,Q^2) \end{aligned} \tag{4}$$

Thus, the FFs cancel and Δu_V and Δd_V are expressed solely in terms of measurable quantities and the known unpolarized u_V and d_V. Note that the measurable quantities (l.h.s.) depend, in general, on (x,z,Q^2), while the LO expressions (r.h.s.) depend on (x,Q^2) only. This can serve as a test for the LO approximation or as an estimate of the theoretical systematic error when applying LO.

When going from LO to NLO, the principle difference is that the simple products are replaced by convolutions and that gluons enter the cross sections. As explained above, in $A_N^{\pi^+-\pi^-}$ the gluon contributions always drop out and we obtain:

$$\begin{aligned} A_p^{\pi^+-\pi^-} &= \frac{(4\Delta u_V - \Delta d_V)[1 + \otimes(\alpha_s/2\pi)\Delta C_{qq}\otimes]D_u^{\pi^+-\pi^-}}{(4u_V - d_V)[1 + \otimes(\alpha_s/2\pi)C_{qq}\otimes]D_u^{\pi^+-\pi^-}} \\ A_n^{\pi^+-\pi^-} &= \frac{(4\Delta d_V - \Delta u_V)[1 + \otimes(\alpha_s/2\pi)\Delta C_{qq}\otimes]D_u^{\pi^+-\pi^-}}{(4d_V - u_V)[1 + \otimes(\alpha_s/2\pi)C_{qq}\otimes]D_u^{\pi^+-\pi^-}}. \end{aligned} \tag{5}$$

The FF $D_u^{\pi^+-\pi^-}$, that enters (5), can be determined in unpolarized SIDIS:

$$R_p^{\pi^+-\pi^-} = \frac{[4u_V - d_V][1+\otimes(\alpha_s/2\pi)C_{qq}\otimes]D_u^{\pi^+-\pi^-}}{18F_1^p\,[1+2\gamma(y)\,R^p]}$$
$$R_n^{\pi^+-\pi^-} = \frac{[4d_V - u_V][1+\otimes(\alpha_s/2\pi)C_{qq}\otimes]D_u^{\pi^+-\pi^-}}{18F_1^n\,[1+2\gamma(y)\,R^n]}, \tag{6}$$

where $\gamma(y) = (1-y)/[1+(1-y)^2]$.

Recently the HERMES collaboration published [6] very precise data for unpolarized SIDIS for $\pi^\pm$ production, that allows to determine $D_u^{\pi^+-\pi^-}$ directly, without requiring knowledge of the other FFs:

$$D_u^{\pi^+-\pi^-} = \frac{9\,(R_p^{\pi^+} - R_p^{\pi^-})\,\sigma_p^{DIS}}{4\,u_V - d_V}. \tag{7}$$

This was done in [7], where $D_{u,d,s}^{\pi^+}$ were determined separately combining the HERMES and the LEP e^+e^- inclusive data on $\pi^\pm$-production. For the first time the u-quark FFs of the pions were obtained by the EMC collaboration [8].

4. Having thus determined Δu_V and Δd_V we can proceed and determine the SU(2) breaking of the polarized sea quarks. We have:

$$(\Delta\bar{u} - \Delta\bar{d}) = \frac{1}{6}\left[\Delta q_3 + \Delta d_V - \Delta u_V\right], \quad \text{where} \quad \Delta q_3 = (\Delta u + \Delta\bar{u}) - (\Delta d - \Delta\bar{d}). \tag{8}$$

In LO the valence quarks are deterimend via (4) and Δq_3 is determined by $\Delta q_3 = g_1^p(x,Q^2) - g_1^n(x,Q^2)$. In NLO the valence quarks are deterimend via (5) and Δq_3 is obtained by the NLO expression:

$$g_1^p(x,Q^2) - g_1^n(x,Q^2) = \frac{1}{6}\Delta q_3 \otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q\right). \tag{9}$$

As SU(2) breaking is expected to be small, an NLO treatment should be important.

Thus, $A_N^{\pi^+-\pi^-}$ and g_1^N determine Δu_V, Δd_V and $\Delta\bar{u} - \Delta\bar{d}$, , both in LO and NLO, without any assumptions. Note that, due to $D_s^{\pi^+-\pi^-} = 0$, even the commonly used $s = \bar{s}$ or $\Delta s = \Delta\bar{s}$ are not made.

5. If $h = K^\pm$ (K^0 are not measured) we cannot use isospin symmetry. However, if we make the natural assumption that unfavoured transitions for K^+ and K^- are equal (but not small): $D_d^{K^+-K^-} = 0$, then we can use $A_N^{K^+-K^-}$ to determine $(\Delta s - \Delta\bar{s})\,D_s^{K^+-K^-}$. We assume that Δu_V and Δd_V are already determined from

the pion SIDIS. Note that as the strange quark is a valence quark for $K^\pm$, $D_s^{K^+-K^-}$ is not small and $(\Delta s-\Delta\bar{s})\, D_s^{K^+-K^-}$ would be nonzero only if $(\Delta s-\Delta\bar{s})\neq 0$.

In LO we have:

$$\begin{aligned} A_p^{K^+-K^-} &= \frac{4\Delta u_V D_u^{K^+-K^-}+(\Delta s-\Delta\bar{s})D_s^{K^+-K^-}}{4u_V D_u^{K^+-K^-}+(s-\bar{s})D_s^{K^+-K^-}} \\ A_n^{K^+-K^-} &= \frac{4\Delta d_V D_u^{K^+-K^-}+(\Delta s-\Delta\bar{s})D_s^{K^+-K^-}}{4d_V D_u^{K^+-K^-}+(s-\bar{s})D_s^{K^+-K^-}} \end{aligned} \tag{10}$$

We determine $D_u^{K^+-K^-}$ and $(s-\bar{s})D_s^{K^+-K^-}$ from unpolarized SIDIS:

$$\begin{aligned} R_p^{K^+-K^-} &= \frac{4\,u_V\,D_u^{K^+-K^-}+(s-\bar{s})D_s^{K^+-K^-}}{18F_1^p} \\ R_n^{K^+-K^-} &= \frac{4\,d_V\,D_u^{K^+-K^-}+(s-\bar{s})D_s^{K^+-K^-}}{18\,F_1^n}. \end{aligned} \tag{11}$$

Following the same argument as above, these ratios will shed light on the $(s-\bar{s})$ difference.

In NLO the same quantities enter and the expressions are analogous to (10)-(11), where the simple products are replaced by convolutions. The full expressions can be found in [4]. Here we present the considerably simpler formulae which result when one takes $s=\bar{s}$:

$$\begin{aligned} A_p^{K^+-K^-} &= \frac{[4\Delta u_V D_u^{K^+-K^-}+(\Delta s-\Delta\bar{s})D_s^{K^+-K^-}]\,[1+\otimes(\alpha_s/2\pi)\Delta C_{qq}\otimes]}{4\,u_V\,[1+\otimes\,(\alpha_s/2\pi)\,C_{qq}\otimes]\,D_u^{K^+-K^-}} \\ A_n^{K^+-K^-} &= \frac{[4\,\Delta d_V D_u^{K^+-K^-}+(\Delta s-\Delta\bar{s})D_s^{K^+-K^-}]\,[1+\otimes(\alpha_s/2\pi)\Delta C_{qq}\otimes]}{4\,d_V\,[1+\otimes\,(\alpha_s/2\pi)\,C_{qq}\otimes]\,D_u^{K^+-K^-}} \end{aligned} \tag{12}$$

From unpolarized SIDIS we determine $D_u^{K^+-K^-}$:

$$\begin{aligned} R_p^{K^+-K^-} &= \frac{2u_V\,[1+\otimes\,(\alpha_s/2\pi)C_{qq}\otimes]\,D_u^{K^+-K^-}}{9F_1^p\,[1+2\gamma(y)\,R^p]} \\ R_n^{K^+-K^-} &= \frac{2d_V\,[1+\otimes\,(\alpha_s/2\pi)\,C_{qq}\otimes]\,D_u^{K^+-K^-}}{9F_1^n\,[1+2\gamma(y)\,R^n]}. \end{aligned} \tag{13}$$

6. If $h=\Lambda,\bar{\Lambda}$, then SU(2) invariance implies $D_u^{\Lambda-\bar{\Lambda}}=D_d^{\Lambda-\bar{\Lambda}}$ and no conditions on $D_s^{\Lambda-\bar{\Lambda}}$. Thus, $A_N^{\Lambda-\bar{\Lambda}}$ will give information on $(\Delta s-\Delta\bar{s})$ without any assumptions. In LO we have:

$$\begin{aligned} A_p^{\Lambda-\bar{\Lambda}} &= \frac{(4\Delta u_V+\Delta d_V)D_u^{\Lambda-\bar{\Lambda}}+(\Delta s-\Delta\bar{s})D_s^{\Lambda-\bar{\Lambda}}}{(4u_V+d_V)D_u^{\Lambda-\bar{\Lambda}}+(s-\bar{s})D_s^{\Lambda-\bar{\Lambda}}} \\ A_n^{\Lambda-\bar{\Lambda}} &= \frac{(4\Delta d_V+\Delta u_V)D_u^{\Lambda-\bar{\Lambda}}+(\Delta s-\Delta\bar{s})D_s^{\Lambda-\bar{\Lambda}}}{(4d_V+u_V)D_u^{\Lambda-\bar{\Lambda}}+(s-\bar{s})D_s^{\Lambda-\bar{\Lambda}}}. \end{aligned} \tag{14}$$

Unpolarized SIDIS could determine $D_u^{\Lambda-\bar{\Lambda}}$ and $(s-\bar{s})D_s^{\Lambda-\bar{\Lambda}}$:

$$\begin{aligned} R_p^{\Lambda-\bar{\Lambda}} &= \frac{(4\,u_V + d_V)D_u^{\Lambda-\bar{\Lambda}} + (s-\bar{s})D_s^{\Lambda-\bar{\Lambda}}}{18F_1^p} \\ R_n^{\Lambda-\bar{\Lambda}} &= \frac{(4\,d_V + u_V)D_u^{\Lambda-\bar{\Lambda}} + (s-\bar{s})D_s^{\Lambda-\bar{\Lambda}}}{18\,F_1^n}. \end{aligned} \tag{15}$$

The expressions in NLO are straightforward and can be found in [4].

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